Math CAMPPP 2011

Plenary 2

Responding to Student Thinking In the Moment

Why Work With Patterns?

- Accessible to most students, K-12
- Because it works: Students who use linear growing patterns, diagrams, and visual representations when considering algebraic relationships are more successful at understanding algebraic relationships, finding generalizations, and providing valid justifications (e.g., Hoyles & Healy, 1999; Lannin et al, 2006; Nuffield Report, 2010 etc etc etc.)

Why Work With Patterns?

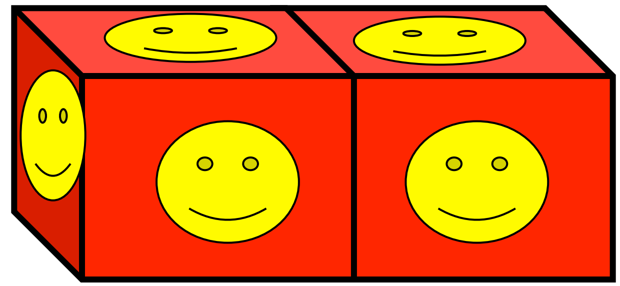
- Offers a dynamic way of considering the relationships between variables
- This can encourage reflection on the concepts underlying the notation that students use, potentially reducing the traditional errors they make when using formal algebraic symbols (Bishop, 1997)

Let's Work With Some Patterns!

1. Cube Sticker Problem
2. Matchstick Problem
3. Pine Tree Problem

- Work on all three in succession
- Track your thinking: Is it changing depending on the problem context? From problem to problem?

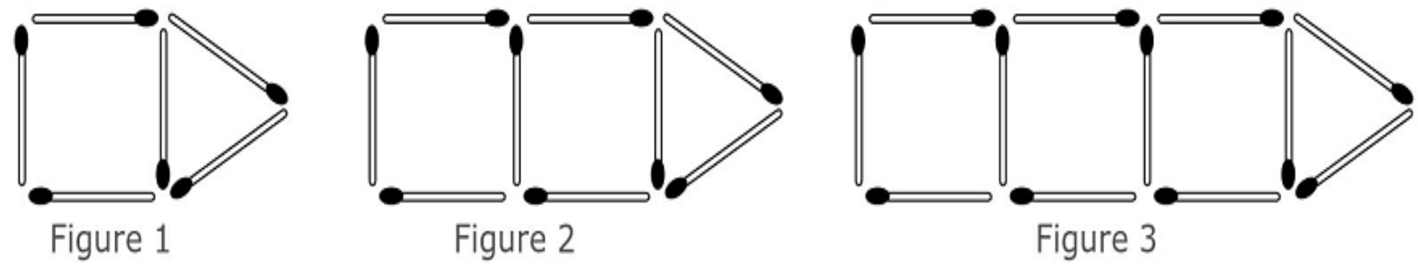
Cube Sticker Problem



A company makes coloured rods by joining cubes in a row and using a sticker machine to place “smiley” stickers on the rods. The machine places exactly one sticker on each exposed face of each cube. Every exposed face of each cube has to have a sticker, so this length-2 rod would need 10 stickers. How many stickers would you need for rods of lengths 1 to 10? Explain how you determined those values.

1. How many stickers would you need for a rod of length 20? Of length 50?
2. Suppose a rod needed 150 stickers. What is the length of the rod? Explain how you determined this.
3. Explain how you could find the number of stickers needed for a rod of *any* length. Write a rule or formula that you could use to determine this.

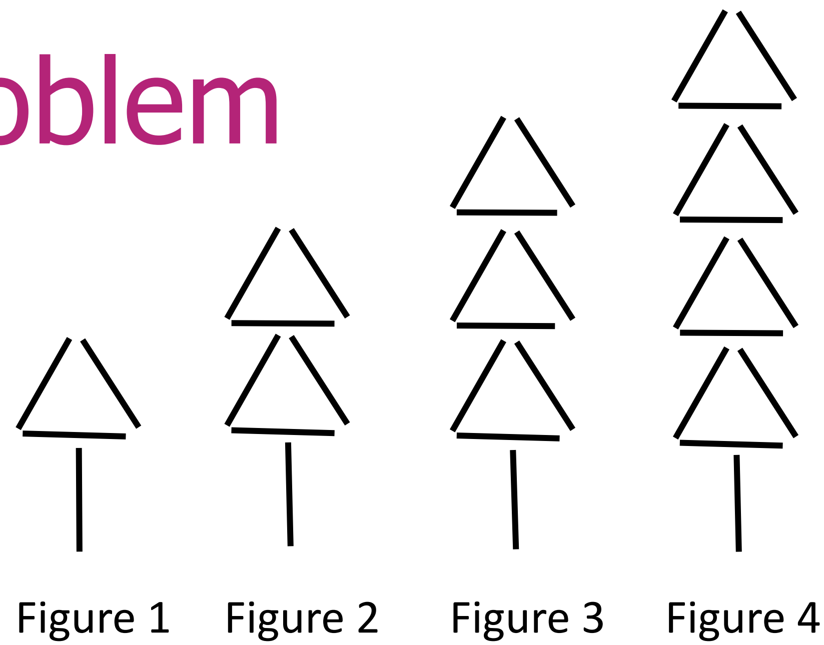
Matchstick Problem



- How many matchsticks would you need for the 10th figure?
- How might you solve this problem a different way?
- Explain how you could find the number of matchsticks for ANY figure.

MathGAINS

Pine Tree Problem



- How many toothpicks would you need for the 10th figure? What about the 100th figure?
- What is the rule for this pattern?
- How do you know your rule works?

Did Your Thinking Change?

- Tell your group what you observed (in yourself and in others)
- May want to discuss this further in your breakout

Generalizing

- Students have difficulty moving from a focus on particular examples toward creating generalizations
- If generalization does not take place, algebra cannot possibly be understood (Dienes, 1961)
- An algebraic rule is a generalization – a mathematical statement that models a situation for any value in the defined domain of the variables

MathGAINS

Types of Generalization

- Recursive generalization:
  - Establishing a general relationship between the nth and n+1 output values (so considering the change in only one set of data – like “add three tiles each time”)
- Explicit generalization:
  - Establishing a general relationship between the input values and their corresponding output values (considering the co-variation, or co-change, of two sets of data – like “tiles = position number x3”)

Generalization and Justification

- Students' justifications provide a window for viewing the degree to which they see the broad nature of their generalizations
- Encouraging students to provide justifications for their rules helps them to see the general relationships that exist in the problem contexts
- It also helps the teacher know how to press students for further understanding (*Key Questions*)

MathGAINS

Generalization Strategies

Strategy	Description
Counting	Drawing a picture or constructing a model to represent the situation to count the desired attributes
Recursive	Building on the previous term or terms in the sequence to determine subsequent terms (Additive thinking)
Whole-object	Using a portion as a unit to construct a larger unit by multiplying. There may or may not be an appropriate adjustment for over-or-undercounting.
Guess-and-check	Guessing a rule without regard to why this rule might work. Usually this involves experimenting with various operations and numbers provided in the problem situation.
Contextual	Constructing an explicit rule that expresses the co-variation of two sets of data, based on information provided in the situation. An explicit rule can allow for the prediction of *any* term number in the pattern.

Let's Look At What Students Do

Counting Strategy

Drawing a picture or constructing a model to represent the situation to count the desired attribute

MathGAINS

Recursive Strategy

Building on the previous term in the sequence to determine subsequent terms.

"Each time you add on 4 more stickers. So for each new cube you just add four more as the length gets bigger by 1 cube."

Whole Object (Proportional Reasoning)

Using a portion as a unit to construct a larger unit by multiplying. There may or may not be an appropriate adjustment for over- or undercounting.

> "If there are 10 stickers for a length-2 rod, then there would be 20 stickers for a length-4 rod."

MathGAINS

Guess and Check Strategy

- Guessing a rule, and finding an answer for *one specific case*, without regard to why this rule might work.
- Also called "local tactics": attempting to find a rule to fit a particular instance of the pattern rather than understanding a general relation in the problem situation.

"For a length-2 rod the rule is cubes times 5, because there are 10 stickers."

MathGAINS

Contextual Strategy

Constructing a rule based on information provided in the situation. The rule works for *any* instance.

"For any length rod, you have to multiply by 4 because there are stickers along each side, and then there are always 2 stickers at the end."

MathGAINS

Justification and Proof

- Justifications are acceptable when they meet the criteria that are established in the mathematical community of the classroom
- The notion of proof depends on the social acceptability of the proof at a given time
- Such guidelines for acceptability are subject to change
- This means that everyone from K to 12 can and should be encouraged to justify their solutions!

Justification Framework

Justification Level	Description
Level 0: No Justification	Responses do not address justification
Level 1: Appeal to external authority	Reference is made to the correctness stated by some other individual or reference material
Level 2: Empirical evidence	Justification is provided through the correctness of <u>particular</u> examples
Level 3: Generic example	Deductive justification is expressed for a particular instance.
Level 4: Deductive justification	Validity is given through a deductive argument that is independent of particular instances

What type of Generalization?
What level of Justification?

Andrew: For 4 cubes, I scratched one of the stickers off the end of the 3-cube rod, which would be 13 stickers, then I added 5 more on because that's how many more stickers I would need which would be 18 stickers. I did it for 6 and it came out 26, and then for 7 it came out 30, and for 8 it came out to 34..and for 10 it was 42.

Teacher: How about for 20 cubes?

Andrew: Twenty? I figured out 42 for 10, so I could do 42 times 2 which would be 84.

What type of Generalization?
What level of Justification?

Gail:	What I did was I multiplied the length of the rod by 4 and then I added 2. So for a rod of 1 I multiplied it by 4, which would be 4 and then I added 2. So 6 stickers on the first cube. Then for the second one I did the same thing, multiplied it by 4 and I got 8 and then I added 2. It was 10 stickers.
Teacher:	How about for 10 cubes?
Gail:	(Pointing to the cubes) There would be 10 there on top, 10 on this side, 10 on this side which is 30 and 10 on the bottom which would be 40. 42, one on each end. So that's my formula, multiplying by 4, and then you add 2 because there's a face there and there on the ends.

Student Discussions

Gail:	What was your way?
Frank:	I minused 2 and multiplied it by 4 and added 10 and it works.
Helen:	Ours was easy. I took it times 4 plus 2.
Gail:	Yeah, that's what I did.
Frank:	Times 4 plus 2?
Helen:	Ya, that works. And I have the same numbers as you, so 10 is 42.

Responding in the Moment

- Key Questions support and facilitate student algebraic reasoning
- How do you know your rule works?
- Does your rule work for any case?
- Tell me your thinking when you worked on this problem
- Can you test your rule another way?

Student Transcripts

- Look at the five transcripts: The Pine Tree Problem
- At your table, discuss what the students are doing/thinking in each case (Generalization strategies? Level of justification?)
- What would you say next to each of these students? (Responding in the Moment)
- Discussion important at your table because we won't have time to take this up whole group

What is Nicole Doing?

Ruth: You drew out just the one picture of the tenth one. How did you know that that is what the tenth one would look like?

Nicole: Because in figure one there is only one square and a triangle, and in figure two there is two squares and a triangle, and in figure three there is three. So I knew that in figure ten there would be ten squares and a triangle.

Ruth: And then you counted all the lines?

Nicole: The toothpicks…

[…]

Ruth: Ok so there are 33 for the tenth one, how many would there be if you built it to the 100th one?

Nicole: One hundred and thirty-three.

Ruth: Why do you think there would be one hundred and thirty-three?

Nicole: I don't know.

What is Miranda Doing?

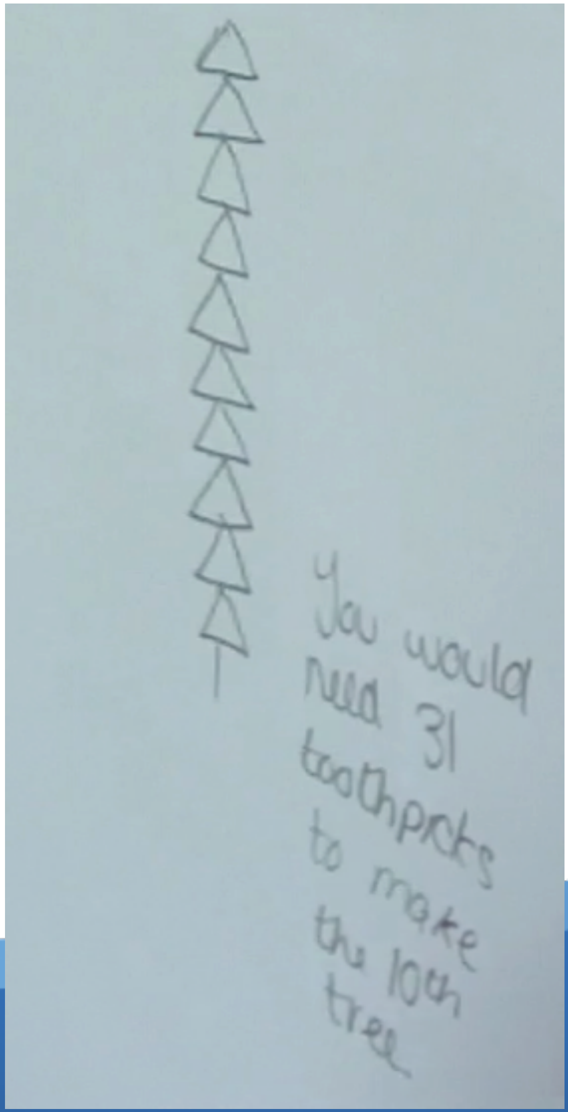
Miranda: Okay so I just kept adding three for the tenth…well you just keep adding three toothpicks to make the tenth figure. And so I just drew it out and…

Ruth: Can you show me how you're counting? Can you count out loud so I can see?

Miranda: Three, six, nine, twelve, fifteen, eighteen, twenty-one, twenty-four, twenty-seven, thirty plus one is thirty-one.

Ruth: So you counted by three's?

Miranda: Yes.



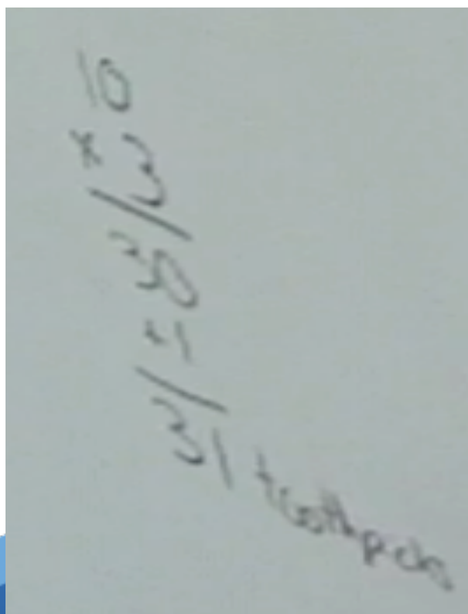
What is Paige Doing?

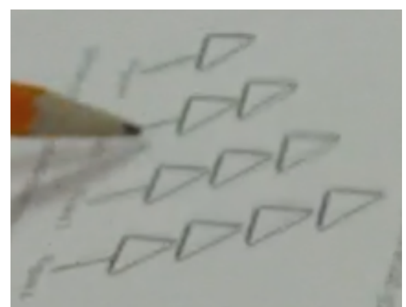
Ruth: Paige, what did you do?

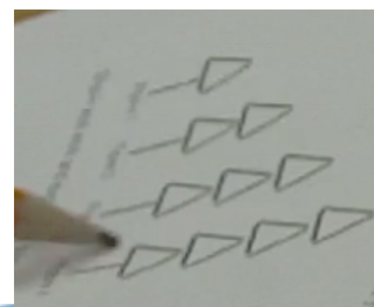
Paige: Um, well I found the pattern rule. I found that for the tenth tree is times three plus one, so I did ten times three is thirty and then I added one and it's thirty-one.

Ruth: So how did you know it was times three plus one?

Paige: Cause there's three (pointing to triangle) and then there's one for each (pointing to the trunk for each tree). Yeah.







What are Liam and Luke Doing?

Liam: Okay well I realized that at figure 1 you have one as the base, so I took that as the number that doesn’t change. And you have three toothpicks so I figure it’s position number times three plus one. So then I go up to here (second figure) and the same thing works cause three times two is six and one is seven. So I just went to the tenth, which is ten times three, which is thirty plus one is thirty-one.

Luke: I just saw that each time it increased by three and initially it had four, so if I went back one then subtract three it would be… the first one would only have one toothpick so I knew that would be plus one and then I multiplied by three each time. So I determined the rule would be times three plus one and then I just input the tenth position so ten times three plus one, thirty plus one, thirty one.

Ruth: What if it was the one hundredth?

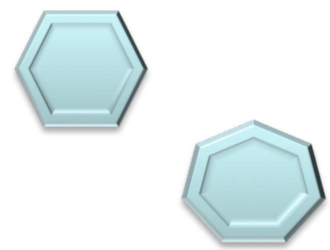
Luke: The one hundredth, I would just do it for one hundred it would be one hundred times three, three hundred plus one would be three hundred and one.

Responding in the Moment

- Math-talk in class: encourage justifications of generalizations
  - Encourage explanations using generic examples
  - Encourage justifications with respect to how calculations relate to the students' interpretation of the problem context
- Activities that allow for students to connect rules with visual representations increase the likelihood of student success
- Discussions about the limitations of focusing on particular values

Furthering Student Thinking

- Variation – hold one variable the same and vary the other
- For example, in the cube sticker problem, what if the rods were made of
  - Hexagonal prisms?
  - Octagonal prisms?



- What is varied? What stays the same?
- If students understand the connection between their rule and the original problem context, then this understanding can easily be transferred to *new* situations